ON SOME ASTRONOMICAL PAPYRI AND RELATED PROBLEMS OF ANCIENT GEOGRAPHY

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F. E. Robbins published in 1927 "A new astrological treatise" which is now contained in the third volume of the "Michigan Papyri" [1] as No. 149. This interesting text gives us (in addition to other material) information about the ἀναφοραί, the time of oblique ascension of the zodiacal signs, a concept the importance of which for the ancient theory of the "climata" was first fully understood by E. Honigmann.[2] The same author felt the close relationship between the Greek methods of dealing with this problem and certain features in the Babylonian astronomy of Seleucid times,[3] but, misled by some incomplete or even erroneous information, he did not quite succeed in establishing this relationship to its full extent. I intend in this paper to complete these investigations as far as Greek astronomy is concerned.[4]

The same problem of the ἀναφοραί is dealt with in the Pap. Michigan III, 151, a fact not recognized by the editors because of the bad mutilation of this fragment. I shall give a restoration and commentary of this text as far as its state of preservation permits.

A third astronomical fragment, Pap. Michigan III, 150, refers to the moon. A few more details which still can be gathered from this text might be of interest for the restoration of manuscripts of the same kind.

The notation applied in the following for writing numbers and fractions is consistently sexagesimal: the sign ";" separates integers from fractions and the sign "," different powers of sixty. Thus 1,30 means 90, but 1;30 represents $1\frac{30}{60} = 1\frac{1}{2}$.

Finally, I must express my warmest gratitude to Dr. F. E. Robbins and Dr. H. C. Youtie for frequent careful investigation of the originals in the papyrus collection of the University of Michigan.

§ 1. Pap. Mich. III, 150

1. The content of this small "badly preserved" fragment is given by the following transliteration: [5]

1.	] D	6;[.]^h	♓	7;21	[I]
	] N	2;40	♈	6;35	II
	] N	12;0	♉	6;6	[II or III]
	]9 D	11;6	♊	5;56	[III or IV]
5.	] D	0;20	♋	5;3	IV
	]7 N	[.];10	♌	5;40	[V]
	] D	[.;.]	♍	5;2	VI
	] N	[.;.]	♎	4;4	VII
	] D	11;0	♏	2;49	VIII
10.	] D	3;20	♐	1;19	IX
	] N	2;20	♐	29;21	X
	] D	5;20	♑	27;55	XI
	] N	6;[.]	♒	26;43	[XII]

D and N here mean "day" and "night" respectively, ^{h} stands for ὥρα and the Roman numerals represent the Egyptian months in their regular order, "I" corresponding to "Thoth," "II" to "Phaophi" etc. No left hand or right hand margin is preserved.

2. First, we shall prove the correctness of the conjecture of the editor that "the purpose of the table was to give the positions of the sun on the days of successive new (or full) moons," although with the slight modification that the positions do not refer to the sun but to the moon in opposition to the sun, or in other words, to the full moons.

This proof is very simple. Twelve steps lead from ♓ 7;21 to ♒ 26;43, thus covering 5,49;22 degrees. One step corresponds therefore to 5,49;22 : 12 = 29;6,50°. On the other hand, on the average the sun travels daily ca. 0;59,9° and one synodical month is about 29;32 days long.[6] The average path of the sun during one synodic month is therefore $0;59,9 \cdot 29;32 \approx 29;6,54°$ or almost exactly the amount derived from our text.[7]

In order to determine whether the text refers to new moons or full moons, we only have to consider the change in the distances from month to month as given by the text:

[1] See Robbins [1] in the Bibliography at the end.

[2] Honigmann [1].

[3] M. P. III p. 312 ff.

[4] For the general importance of this problem see Neugebauer [1].

[5] I owe the following additions to the edition in M. P. III to Dr. H. C. Youtie: line 2: θ[ωθ] is an error in reading, the papyrus has φ[αωφι]; line 4:]θ̣̄; line 6: ε̣̄ or ζ̣̄ or θ̣̄ (as shown in the text ζ̣̄ is the correct reading); line 9: read φα̣[ρμουθι] in place of [φαρμουθι]; line 13: read ν̣υ̣κτ̣(ος) instead of [...] and Με̣[σορη] in place of [Μεσορη].

[6] The Pap. Michigan III, 149 gives for the sun's velocity 0;59,8,16° per day but Babylonian astronomy from Seleucid time uses 0;59,9 (cf. e.g. Schnabel [1] p. 39, but this paper contains many errors in the commentary as well as in details and in the main thesis). For the length of the synodic month see e.g. Geminus, cap. 18 (ed. Manitius p. 200 ff.).

[7] It is a lapsus calami of the editor when he says (p. 118) that the sun travels "an average of somewhat less (!) than 29° during the moon's synodic period."

	text	difference
1.	♓ 7;21	
	♈ 6;35	29;14
	♉ 6;6	29;31
	♊ 5;56	29;50
5.	♋ 5;3	29;7
	♌ 5;40	30;37
	♍ 5;2	29;22

	text	difference
	♎ 4;4	29;2
	♏ 2;49	28;45
10.	♐ 1;19	28;30
	♐ 29;21	28;2
	♑ 27;55	28;34
	♒ 26;43	28;48

This shows immediately that the movement during the first six months is faster than during the second part of the year, having a secondary maximum in ♊, its minimum in ♐. However, at the beginning of our era the sun's velocity has its minimum in ♊,[8] its maximum in ♐, or in other words just at the points opposite to the places given in the text. This is only possible if the text does not refer to the conjunctions of the moon and the sun but to the oppositions, i.e. to the full moons.[9]

This conclusion is furthermore confirmed by the following consideration. The second column of the text gives Egyptian months from I to XII.[10] Therefore it is highly probable that the first position given in the first column corresponds to the beginning of the year. But in Roman times the sun never stands in the sign of ♓ at the beginning of the Egyptian-Hellenistic calendar, but just opposite in ♌ or ♍. Again only full moons agree with the positions given in the text.

3. After having established the general character of the text we now discuss the dates given in the first part of the first column. The hours of "day" or "night" are abbreviated in the papyrus by ωρᴸ. Unfortunately it seems that this symbol is unknown from elsewhere. It might mean ὥρα or perhaps more precisely ὥρα ἰσημηρίνη because ὥρα alone would be rather inconvenient in an astronomical table, leaving it open whether these "hours" should be interpreted as seasonal hours or as equinoctial hours. For the sake of simplicity we shall assume the latter case, which is at any rate sufficient to restore roughly the gaps in this part of the papyrus, as shown in the following table:

	text		= day	difference
1.	D	6;[.]^h	6;[.]^h	
	N	2;40	14;40	8;[.]^h
	N	12;0	24;0	9;20
	D	11;6	11;6	11;6
5.	D	0;20	0;20	13;14
	N	[3;]10	[15;]10	[14;]50
	D	[6;.]	[6;.]	[15;.]

	text		= day	difference
	N	[8;.]	[20;.]	[14;.]
	D	11;0	11;0	[15;.]
10.	D	3;20	3;20	16;[.]
	N	2;20	14;20	11;0
	D	5;20	5;20	15;0
	N	6;[.]	18;[.]	13;[.]

The comparison of the first and the last line of this table shows a time difference of [354 days] + 12 hours, which agrees with the average length of twelve months (of 29.53 days each) i.e. 354 days + 9 hours.

It is interesting that the reexamination of the papyrus by Prof. Youtie gave in lines 4 and 6 remainders of the dates which must have preceded the hours. In line 4]θ̄ is visible i.e. 9 or 19 or 29. The traces in line 6 could be read as θ̄, ζ̄ or ε̣̄. The last possibility is excluded by the fact that the date of the full moon cannot drop from the 9th to 5th during two months only. Assuming morning epoch in the counting of dates we should get as time difference from month *m* day 9 daytime 11;6 in line 4 to month *m* + 2 day 9 night 3;10 in line 6 a difference of 60 days and 4;4^h, which is too much for two months. If we however assume evening epoch, then this difference becomes $59^d + 4;4^h$ only. On the other hand the reading 7 in line 6 in the case of morning epoch gives a time interval of $58^d + 4;4^h$; in the case of evening epoch however $57^d + 4;4^h$ only, which is too short a time. In other words, the reading 9 requires the assumption of evening epoch, the reading 7 in line 6 requires morning epoch. The existence of the evening epoch in Egyptian dates was strongly contested by Sethe against Ed. Meyer.[11] The most plausible restoration is therefore

line 4.	[month *m*	day .]9	day 11;6
5.	[month *m* + 1	day .8]	day 0;20
6.	[month *m* + 2	day .]7	night [3;]10.

4. One could attempt to date this document exactly, using the positions and hours given in the text. There are, however, so many possibilities which fit these positions equally well that a choice could be made only by ascribing to some of the numbers a reliability which cannot be granted because of the general character of the text, even if we disregard the open problem as to which point of the ecliptic should be assumed as origin. Furthermore, it is obvious that the positions recorded are not the result of observations (full moons during day time!) but of some theoretical calculations the exactitude of which we are not able to judge. We must therefore confine ourselves to the restorations given above.

[8] Cf. e.g. Ptolemy, Almagest III, 4 (ed. Heiberg I, p. 237, 9–11) Apogee: ♊ 5;30°.

[9] The maximum in ♌ must reflect the position of the apogee of the moon. The reading εμ in line 6 cannot be replaced by anything else, as Prof. Youtie kindly informed me.

[10] The repetition of II, III or IV is due to the fact that there must always exist years containing 13 full or new moons (cf. e.g. Neugebauer-Volten [1] for the Egyptian moon calendar).

[11] Sethe [1] p. 130 f.

§ 2. General Remarks about the "Anaphorai" and their Measurement

5. We suppose the ecliptic subdivided into twelve parts of 30 degrees each, beginning with the vernal point and proceeding in the direction of the annual movement of the sun. These twelve parts may be called the "zodiacal signs" even when the vernal point does not coincide with the beginning of the sign "aries" but is located for instance on the eighth degree of this sign. We shall emphasize the distinction between the two overlapping subdivisions only when necessary to avoid confusion.

The problem of the "Anaphorai" or oblique ascensions consists in the determination of the time intervals $\alpha_1, \alpha_2, \cdots, \alpha_{12}$ required by the first, second, $\cdots$, twelfth sign to rise above a given horizon. These time intervals are obviously not equally long because of the inclination of the ecliptic to the plane of the daily rotation, the celestial equator. The problem of determining these α's occurs therefore wherever arcs of the ecliptic are to be compared with arcs on the equator—a problem which plays for instance an important rôle in the calculation of the visibility of the new moon, so essential for the oriental moon calendar.

The interest in these magnitudes α_1 to α_{12} lies furthermore in their relation to the determination of the variability of the length of the days during the seasons. This is an immediate consequence of the following consideration. From the fact that ecliptic and horizon are both great-circles on the celestial sphere it follows that, at any moment, one half of the ecliptic is above the horizon,[12] or, in other words, six of the zodiacal signs. On the other hand, the length of a day is the time from sunrise to sunset during which period six signs rose one after the other. These signs compose the semicircle above the horizon at the moment of sunset. This gives the fundamental theorem: *The length of a day corresponding to a certain position of the sun in the ecliptic equals the sum of the rising times for the next 180 degrees of the zodiac.*

In order to express this relation by a short formula, we introduce twelve magnitudes $C_1, C_2, \cdots, C_{12}$ by the following definition: C_1 gives the length of the day corresponding to the sun standing in the vernal point, C_2 the length of the day corresponding to a sun position 30 degrees more advanced and so on to C_{12} when the sun stands at the beginning of the twelfth sign. The theorem which connects the ἀναφοραί and the length of the days is then simply

$$\begin{aligned} C_1 &= \alpha_1 + \alpha_2 + \cdots + \alpha_6 \\ C_2 &= \alpha_2 + \alpha_3 + \cdots + \alpha_7 \\ &\vdots \\ C_{12} &= \alpha_{12} + \alpha_1 + \cdots + \alpha_5 \end{aligned} \tag{1}$$

These relations are essential for the understanding of all ancient discussions of the rising times of the zodiacal signs.

A second system of formulae is equally important, establishing the symmetry of the rising times with respect to the vernal point and the autumn point, namely

$$\begin{aligned} \alpha_1 &= \alpha_{12} & \alpha_4 &= \alpha_9 \\ \alpha_2 &= \alpha_{11} & \alpha_5 &= \alpha_8 \\ \alpha_3 &= \alpha_{10} & \alpha_6 &= \alpha_7 \end{aligned} \tag{2}$$

as can be seen by simple consideration of a sphere. Both relations (1) and (2) are general facts independent of the actual values of the α's.

6. The problem of determining the values of the α's can obviously be considered as a problem of spherical trigonometry and was undoubtedly one of the main causes of the development of this field. We find its complete solution by trigonometrical methods given in Ptolemy's "Almagest" (about 150 A.D.) but it is significant for the importance attributed to this question that one half of Ptolemy's "Planisphaerium"[13] is devoted to another solution by such absolutely different methods as stereographic projection and considerations which we today would call descriptive geometry and nomography. From earlier sources, however, only approximative solutions are testified and these methods we intend to discuss here.[14] The essential feature in this preliminary stage in the determination of the rising times is the assumption that the values of the α's increase or decrease, respectively, by a constant amount d. We call these methods therefore "*linear methods*," in contrast to the exact "trigonometrical methods."[15]

These "linear" methods appear in two slightly different forms. The "strictly" linear method assumes that the increase of the α's from α_1 to α_6 is always d or

$$\begin{aligned} \alpha_2 &= \alpha_1 + d = \alpha_{11} & \alpha_5 &= \alpha_4 + d = \alpha_8 \\ \alpha_3 &= \alpha_2 + d = \alpha_{10} & \alpha_6 &= \alpha_5 + d = \alpha_7 \\ \alpha_4 &= \alpha_3 + d = \alpha_9 \end{aligned} \tag{3a}$$

The "generalized" linear method, however, assumes in the middle of the α-sequence twice as much as elsewhere or

$$\begin{aligned} \alpha_2 &= \alpha_1 + d = \alpha_{11} & \alpha_5 &= \alpha_4 + d = \alpha_8 \\ \alpha_3 &= \alpha_2 + d = \alpha_{10} & \alpha_6 &= \alpha_5 + d = \alpha_7 \\ \alpha_4 &= \alpha_3 + 2d = \alpha_9 \end{aligned} \tag{3b}$$

[12] This statement ignores not only atmospheric refraction but also the movement of the sun in the ecliptic during the day. For the approximative methods discussed in the following, both effects are of no influence.

[13] Ptolemy, opera II p. 225–259. As various remarks in this work prove, it was written after the Almagest (cf. e.g. p. 234, 16, p. 242, 2). It is usually assumed that this method had been invented by Hipparchus; cf. Delambre HAA II p. 454 f. The main source is a letter of Synesius (Migne PG 66 p. 1577, transl. Fitzgerald p. 263); the passage from Proclus, quoted by Delambre II p. 454, seems to be spurious.

[14] The development of the "trigonometrical methods" in Greek astronomy will be discussed in the thesis of Mr. Olaf Schmidt, Brown University.

[15] Ptolemy, Tetrabiblos I, 20 (ed. Robbins p. 94/95) = I, 21 (ed. Boll-Boer p. 46) speaks about "the common method, based upon evenly progressing increases in the ascensions which is not even close to the truth" (transl. Robbins).

In the following, we refer to these two kinds of assumption as "System A" (3a) and "System B" (3b) respectively.

7. The problem of determining the length of day and night from month to month according to the position of the sun in the zodiac is reduced by means of the relations (1) to the determination of the twelve magnitudes α_1 to α_{12}. The symmetry-relation (2) reduces the number of unknown quantities to six. If we furthermore make the assumption that the variation of the α's follows either the scheme A or B, the problem is still more simplified since because of (3) only two quantities, say α_1 and d, remain unknown. Therefore, we need only two independent relations in order to know the length of day and night for any given position of the sun.

Two such relations are furnished by the knowledge of the length M of the longest day and the length m of the shortest day. M occurs when the sun's distance from the vernal point amounts to 90°, i.e. M is the same as C_4. Hence, from (1) and (2) follows

$$(4') \qquad M = 2(\alpha_4 + \alpha_5 + \alpha_6)$$

and correspondingly

$$(4'') \qquad m = 2(\alpha_1 + \alpha_2 + \alpha_3).$$

If we work with "System A," then (3a) gives

$$(5a) \qquad \begin{aligned} M &= 6\alpha_1 + 24d \\ m &= 6\alpha_1 + \ 6d \end{aligned}$$

and (3b) gives correspondingly in the case of "System B"

$$(5b) \qquad \begin{aligned} M &= 6\alpha_1 + 30d \\ m &= 6\alpha_1 + \ 6d. \end{aligned}$$

From (5) both α_1 and d can be easily calculated and this gives us all the α's and C's.

This shows that if we know, for a given place, the length M of its longest day and the length m of its shortest day, then we are able to calculate all α's, from which results the law of the change of the length of the days during the year. Ancient astronomy simplified the problem further by assuming that the length m of the shortest day equals the length of the shortest night, or by assuming $M + m = 24$ hours.[16] The knowledge of M alone is therefore sufficient to characterize the variation of the days during the seasons.[17]

The essential point in this argumentation is the assumption of the law of variation of the α's, according to either system A or B. Both these systems are only approximations of the real law of dependence of the rising times on the sun's longitude, but it is necessary to emphasize that the obtained results are quite satisfactory as far as the resulting values of the lengths of the days are concerned, at least if we take the inaccuracy of ancient time-measurement into consideration.

8. In order to give concrete examples of the preceding general discussion, we shall, at the end of this paragraph, give the values for the α's and C's as derived from cuneiform sources. However, we must first introduce some remarks concerning the units used to measure time.

The fundamental unit is the Sumerian length measure "danna" (called *bēru* in Akkadian), twelve of which correspond to one day's travel.[18] "Danna" usually is translated by "double hour," but without real reason, because this word does not contain any such element as "double" or "hour." Perhaps, the correct correspondence would be "mile." One sixtieth of this unit is called "stadium" by Manilius.[19]

These "miles" are subdivided into 30 parts each, called "uš," i.e. "length."[20] The transfer of these terrestrial units to the sky, according to the rule that twelve danna equal one day, thus creates the subdivison of the celestial equator into $12 \cdot 30 = 360$ parts. The "uš" corresponds therefore to our "degree," Greek *μοῖρα*.[21] In order to unify our notation, we shall express in the following both the α's and the C's consistently in "degrees," whether or not the original source uses degrees or (equinoctial) hours. Babylonian sources never use any other unit, but in Greek astronomy both units appear. The rule of transformation is for instance given in Mich. Pap. III 149:[22] *ἰσχύουσίν δε οἱ ιε̅ χρόνοι α̅ ὥραν ἰσημερίνην.*

We now are prepared to give the values for the *ἀναφοραί* in Babylonian astronomy of Seleucid times. System A is based on the following values:

$$(6a) \qquad \begin{array}{lll} \alpha_1 = \alpha_{12} = 20^\circ & \alpha_4 = \alpha_9 = 32 & \\ \alpha_2 = \alpha_{11} = 24 & \alpha_5 = \alpha_8 = 36 & d = 4 \\ \alpha_3 = \alpha_{10} = 28 & \alpha_6 = \alpha_7 = 40 & \end{array}$$

from which the following lengths of the days result

[16] This is only correct if we again disregard atmospheric refraction which makes m greater than the complement to M.

[17] For explicit formulae, expressing d and α_1 by M see p. 255.

[18] In older literature, names such as KAS–BU and similar words can be found, because the sign "danna" contains two components which can be read as KAS and BU respectively.

[19] Manilius III, 282 ff.

[20] Here too, many misreadings are current in literature, for example geš or *imdu*. Even a non-existing Greek root *στάω* has been introduced in order to explain a wrong Akkadian etymology. Cf. Neugebauer [2] p. 274 note 126.

[21] Hypsicles (I quote according to the text established by V. de Falco for a new edition of the "Anaphorikos" under preparation): *Τοῦ τῶν ζῳδίων κύκλου εἰς τξ̅ περιφερείας ἴσας διῃρημένου, ἑκάστη τῶν περιφερειῶν μοῖρα τοπικὴ καλείσθω· ὁμοίως δὴ καὶ τοῦ χρόνου, ἐν ᾧ ὁ ζῳδιακὸς ἀφ' οὗ ἔτυχε σημείου ἐπὶ τὸ αὐτὸ σημεῖον παραγίγνεται, εἰς τξ̅ χρόνους ἴσους διῃρημένου, ἕκαστος τῶν χρόνων μοῖρα χρονικὴ καλείσθω.* (Cf. ed. Manitius col. 5,25 ff.). Analogously Ptolemy, opera II p. 74,6 ff. (*προθ. τῶν πλαν.* 3): *διαιρεθείσης δε τῆς περιφερείας αὐτοῦ* (sc. of the equator) *εις ἴσα τμήματα τξ̅ καλείσθω τὰ τμήματα ἰδίως χρόνοι.*

[22] XII, 9 f.

according to (1):

$$(7a)\quad \begin{array}{ll} C_1 = 3{,}0^\circ & C_7 = 3{,}0 \\ C_2 = 3{,}20 & C_8 = 2{,}40 \\ C_3 = 3{,}32 & C_9 = 2{,}28 \\ C_4 = M = 3{,}36 & C_{10} = m = 2{,}24 \\ C_5 = 3{,}32 & C_{11} = 2{,}28 \\ C_6 = 3{,}20 & C_{12} = 2{,}40. \end{array}$$

System B assumes

$$(6b)\quad \begin{array}{lll} \alpha_1 = \alpha_{12} = 21^\circ & \alpha_4 = \alpha_9 = 33 & \\ \alpha_2 = \alpha_{11} = 24 & \alpha_5 = \alpha_8 = 36 & d = 3 \quad 2d = 6 \\ \alpha_3 = \alpha_{10} = 27 & \alpha_6 = \alpha_7 = 39 & \end{array}$$

and thus

$$(7b)\quad \begin{array}{ll} C_1 = 3{,}0^\circ & C_7 = 3{,}0 \\ C_2 = 3{,}18 & C_8 = 2{,}42 \\ C_3 = 3{,}30 & C_9 = 2{,}30 \\ C_4 = M = 3{,}36 & C_{10} = m = 2{,}24 \\ C_5 = 3{,}30 & C_{11} = 2{,}30 \\ C_6 = 3{,}18 & C_{12} = 2{,}42. \end{array}$$

For sun positions at arbitrary points between the boundaries of the zodiacal signs, linear interpolation is adopted. This holds for all the lists discussed in the following.

The values C in (7a) and (7b) were discovered by Father Kugler in cuneiform tablets concerning the calculation of the new and full moons.[23] He established at the same time that system B is of later date than system A,[24] a statement which has been supported by recent investigations.[25]

As stated above, knowledge of the magnitude M is sufficient to determine all the α's, after the single assumption of the "system" to be used. If we express all units in degrees, then $M + m$ becomes 6,0° and from (5) p. 254 it follows that

$$(8)\qquad d = \begin{cases} \frac{1}{9}(M - 3{,}0) & \text{system A} \\ \frac{1}{12}(M - 3{,}0) & \text{system B} \end{cases}$$

and

$$(9)\qquad \alpha_1 = \begin{cases} 1{,}20 - \frac{5}{18}M & \text{system A} \\ 1{,}15 - \frac{1}{4}M & \text{system B} \end{cases}$$

from which all further α's can be calculated.

§ 3. The Anaphorai in Pap. Mich. III, 149

9. The two Babylonian schemes for the length of the days, as given in the preceding section, agree as to the extremal values $M = 3{,}36$ and $m = 2{,}24$; that is, they have the ratio $M : m = 3 : 2$ in common. This shows that they both refer to the same geographical latitude. The investigation of astronomical theories contained in the cuneiform tablets of Seleucid times does not reveal that Babylonian astronomy extended the scheme for the C's and α's to different latitudes. Until proof of the contrary, we may therefore assume that the idea of introducing variable geographical latitudes originated in Greek astronomy.

The word "latitude," however, must not be taken in the strict modern sense of one of the spherical coordinates on the earth. The geographical latitude does not occur in the Greek tables of obliquascensional times until they reached their final form, based on spherical trigonometry, as given in Ptolemy's Almagest, book II, chapter 8. But even these tables are not compiled with respect to linear increase in geographical latitude, but with respect to the maximum length of the day (our M), which increases in Ptolemy's tables by the constant amount of one half hour.[26] In the Geography we are instructed to draw the παραλλῆλοι in such a way that M increases from the equator to the 14th parallel ($\varphi = 45$) by the constant amount of $\frac{1}{4}$ of an hour, from the 15th to the 19th by $\frac{1}{2}$ hour and by one hour between the three last ones.[27] This obviously reflects the older use of the non-trigonometrical or "linear" tables which we now proceed to consider.

The Mich. Pap. III, 149 contains the rising times for Aries and Libra (i.e. α_1 and α_6 in our notation) for seven different locations,[28] the so-called "seven climata." The content of the text [29] can be condensed in the following list:

[23] Kugler [1].

[24] Unfortunately called by him in reversed order "System II" and "System I" respectively. A more exact date of the origin than "in the beginning of the Seleucid period" cannot be given. Cf. p. 262 note 61.

[25] Neugebauer [2]. A third scheme has been proposed by Kugler, but is undoubtedly wrong, as Schnabel [1] p. 32 f. remarked and as can be proved by consequences in the calculation of the moon's crescent. Honigmann's discussion M. P. III p. 317 therefore has no basis whatsoever.

[26] Seven climata in Almagest II, 13 and VI, 11 (opera I p. 174 ff. and p. 538/9) and in the Analemma (opera II p. 282, 17 ff. cf. also p. 160, 2 ff.), only five in the φάσεις (opera II p. 4).

[27] Ptolemy, Geogr. I, 23. For a modern treatment cf. Mžik [1].

[28] In the preceding section I propose the following restoration in XI, 32: αἰ̣θ[η]ορη̣[τος].

[29] M. P. III, 149, col. XI, 40 to XII, 6.

(10)

climata	1 Ethiopia	2 Syria	3 Rhodes	4 Asia, Ionia	5 Argos	6 Rome, Italy, Marit. Gaul	7 [Asia], Germany, Britain [30]
α_1	22	21	20	19	18	17	16
α_6	38	39	40	41	42	43	44
d	2;40	3	3;20	3;40	4	4;20	4;40

The numbers d are mentioned as the "προσθέσεις" or "ἀφαιρέσεις" (amounts to be added or subtracted) in order to complete the table for each clima. Unfortunately, there is one more sentence [31] ἐν παντὶ οὖν κλίματι ὁ καρκίνος καὶ ὁ αἰγόκερος ἐν λ̄ χρόνοις ἀνέχθησαν i.e. for all latitudes $\alpha_4 = \alpha_{10} = 30°$. The editors therefore reconstructed the tables for the α's in such a way that $\alpha_4 = \alpha_{10}$. For example, for the second clima this resulted in the following table

(11)
$$\begin{aligned} \alpha_1 &= 21 \\ \alpha_2 &= 24 = \alpha_{12} \\ \alpha_3 &= 27 = \alpha_{11} \\ \alpha_4 &= 30 = \alpha_{10} \\ \alpha_5 &= 33 = \alpha_9 \\ \alpha_6 &= 36 = \alpha_8 \\ \alpha_7 &= 39 \end{aligned}$$

and correspondingly for the other climata. These tables preserve the extreme values and the differences as given by the text but differ from the text by assuming that the maximum value refers to α_7 instead of to α_6. Not only do these tables not fulfill the fundamental relations (2), but also the corresponding lengths of the days become impossible. For example, the "equinoctial" day should be, according to definition,

$$C_1 = C_7 = 3,0 = \alpha_1 + \alpha_2 + \cdots + \alpha_6.$$

Using the above assumed numbers, it is only 2,52, and the resulting "equinox" has unequal length of day and night!

There seems to me to be no reason for assuming that such obvious contradictions should have escaped any ancient astronomer. Therefore we must construct the tables from the numbers given and thereafter discuss the meaning of the sentence in question. If we proceed in this way, we can easily give a perfectly correct scheme for the α's in seven climata by use of the elements (10) given in the papyrus. We need only remark that the given differences, together with the values of α_1 and α_6, make it necessary to insert $2d$ somewhere, or in other words, to use system B instead of system A. This gives the following scheme:

(12)

climata	1	2	3	4	5	6	7
$\alpha_1 = \alpha_{12}$	22	21	20	19	18	17	16
$\alpha_2 = \alpha_{11}$	22;40	24	23;20	22;40	22	21;20	20;40
$\alpha_3 = \alpha_{10}$	27;20	27	26;40	26;20	26	25;40	25;20
$\alpha_4 = \alpha_9$	32;40	33	33;20	33;40	34	34;20	34;40
$\alpha_5 = \alpha_8$	35;20	36	36;40	37;20	38	38;40	39;20
$\alpha_6 = \alpha_7$	38	39	40	41	42	43	44
d	2;40	3	3;20	3;40	4	4;20	4;40

The numbers of the second clima are identical with the numbers already known from (6b) p. 255. This arrangement not only makes the assumption of serious errors unnecessary, but also reveals a perfectly clear structure: Deriving the C's from it in the usual way, we obtain for the shortest and longest days:

(13)

climata	1	2	3	4	5	6	7
m	2,28	2,24	2,20	2,16	2,12	2,8	2,4
M	3,32	3,36	3,40	3,44	3,48	3,52	3,56
$M : m$	$\frac{53}{37}$	$\frac{3}{2}$	$\frac{11}{7}$	$\frac{28}{17}$	$\frac{19}{11}$	$\frac{29}{16}$	$\frac{59}{31}$

[30] The restoration Ἀσ[ιαι] in XII, 5 seems to me the only possible one in spite of the larger space available (ca. 5 letters destroyed) and the appearance of "Asia" in the fourth column.

[31] M. P. III, 149, XII, 7 ff.

This shows that the climata are arranged in arithmetical progression for m and M. Furthermore, the second clima "Syria" is distinguished by having the simple relation $M : m = 3 : 2$ exactly as Babylon. In other words: the list (12) takes the second clima as being equivalent to Babylon and starting from this latitude, extends the scheme to north and south in such a way that the longest days vary in arithmetic progression.

This scheme is in itself so simple and convincing that I have no doubt that it represents the right interpretation of the text. We can, however, deduce still more arguments from the text itself to show that system B and the latitude Babylon–Syria constitute the basis for this list. The papyrus mentions in XI, 28 ff. that the vernal point ought to be placed at the eighth degree of Cancer and this is exactly the same definition as in the later Babylonian theory ("B"). Furthermore, it is said in XII, 22 ff. and XII, 32 that the day is 14 hours when the sun is in Gemini. According to (1), we get for this magnitude and for the second clima from (12)

$$C_3 = \alpha_3 + \alpha_4 + \cdots + \alpha_8 = 3{,}30^\circ = 14^h$$

exactly as the text. Assuming (11), one would obtain $C_3 = 3{,}21^\circ = 13;24^h$. For the sun in Taurus the text gives in XII, 20 the length of the day as 13 hours, where (12) results in $C_2 = 3{,}18^\circ = 13;12^h$ (cf. p. 253), but (11) in $C_2 = 3{,}9^\circ = 12;36^h$. In the following, we shall meet different instances where fractional parts are simply disregarded; then again 13^h is the result only from (12) but not from (11).

It remains to return once more to the questionable sentence that "in every clima Cancer and Capricornus ascend during 30 *χρόνοι*." We have, I think, two possibilities: either we consider this sentence as the erroneous addition of some scribe, or we take into consideration that the papyrus puts the vernal point at the eighth degree of Cancer. Under this assumption, the arcs of 30 degrees, to which the α's refer, overlap the zodiacal "signs." "Cancer" belongs then partly to α_3, partly to α_4 and "Capricornus" both to α_9 and α_{10}. If we now look at the scheme (12), one could say that a sign which lies "between" α_3 and α_4 or α_9 and α_{10} corresponds for every climata to the average value of 30°, represented in (12) by the dotted line. The sentence in question would therefore be fully correct if the vernal point lay in the 15th degree of the first "sign," but also remains at least intelligible as a loose expression under the present conditions. This explanation remains, of course, open to discussion, but it is certainly unnecessary to derive a contradictory scheme from this single casual remark.

10. Having established the scheme (12) as the method of determining the α's and C's according to system B and Babylon as the basic latitude, we now shall show that this method is in perfect agreement with other Greek sources.

System B is applied by Cleomedes [32] in his law for the variability of the lengths of the days at the Hellespontic clima,[33] which is characterized by $M : m = 5 : 3$.[34] The same rule is given by Martianus Capella [35] (end of 4th cent.) and Gerbert [35] (the pope Silvester II, end of 9th cent.). The resulting α's are

$$\begin{array}{llll} & \alpha_1 = \alpha_{12} = 18;45^\circ = 1;15^h & & \\ & \alpha_2 = \alpha_{11} = 22;30^\circ = 1;30 & & \\ & \alpha_3 = \alpha_{10} = 26;15^\circ = 1;45 & & \\ (14) & \text{- - - - - - - - - - - - - - - - - -} & & d = 3;45^\circ = 0;15^h \\ & \alpha_4 = \alpha_9 \ = 33;45^\circ = 2;15 & & \\ & \alpha_5 = \alpha_8 \ = 37;30^\circ = 2;30 & & \\ & \alpha_6 = \alpha_7 \ = 41;15^\circ = 2;45\ . & & \end{array}$$

System A is represented four times. The most important text is Vettius Valens. He first [37] gives the complete system of the α's exactly as in the Babylonian sources of system A (see p. 254 formula (6a) and (7a)). Then he gives the list of the "seven climata," [38] but now starting from Alexandria [39] (characterized by $M : m = 7 : 5$ [40]) and defining the "climata" such that the M's are in arithmetical progression. The rising times are:

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[32] Cleomedes I, 6 ed. Ziegler p. 50.

[33] I believe to have shown that Cleomedes lived in Lysimachia on the Hellespont (Neugebauer [3]). These passages were interpreted as an arbitrary numerical example by Pogo [1] p. 413 f.

[34] That the longest day at the Hellespont was supposed to be 15 hours is well known; cf. e.g. Ptolemy, Almagest II, 8 (ed. Heiberg p. 138).

[35] Martianus Capella VIII,877 f. ed. Eyssenhardt p. 326 f., ed. Dick p. 462 f.

[36] Gerbert, opera, ed. Bubnow p. 39 f. (Gerbert omits one line).

[37] Vettius Valens I, 7 ed. Kroll p. 23.

[38] Vettius Valens I, 7 ed. Kroll p. 24, 8 ff.

[39] I do not see any "unlösbaren Widerspruch" as Honigmann does (M. P. III p. 302) in the fact that an author uses first one place (Babylon) for a special example of his method and thereafter starts the general list of the climata from another point (Alexandria). This reflects merely the historical facts that the method originated in and for Babylon only and that the scheme of the seven climata was a Hellenistic invention.

Bishop George, who lived around 700 A.D. in Mesopotamia, does exactly the same in giving the rising times according to system A for Babylon, the seven climata, however, according to the scheme of Almagest II, 13 (cf. above p. 255, note 26) which does not contain Babylon (Ryssel [1] p. 48 and p. 49).

[40] E.g. Hypsicles: *ὑποκείσθω δὴ τὸ ἐν Ἀλεξανδρείᾳ τῇ πρὸς Αἴγυπτον κλίμα ἐν ᾧ ἡ μακροτάτη ἡμέρα πρὸς τὴν βραχυτάτην ἡμέραν λόγον ἔχει ὃν ξ̄ πρὸς ε̄* (cf. ed. Manitius col. 7, 3 ff.).

(15)

clima	1	2	3	4	5	6	7
$\alpha_1 = \alpha_{12}$	21;40	20;33,20	19;26,40	18;20	17;13,20	16; 6,40	15
$\alpha_2 = \alpha_{11}$	25	24;20	23;40	23	22;20	21;40	21
$\alpha_3 = \alpha_{10}$	28;20	28; 6,40	27;53,20	27;40	27;26,40	27;13,20	27
$\alpha_4 = \alpha_9$	31;40	31;53,20	32; 6,40	32;20	32;33,20	32;46,20	33
$\alpha_5 = \alpha_8$	35	35;40	36;20	37	37;40	38;20	39
$\alpha_6 = \alpha_7$	38;20	39;26,40	40;33,20	41;40	42;46,40	43;53,20	45
d	3;20	3;46,40	4;13,20	4;40	5; 6,40	5;33,20	6

from which follows

(16)

clima	1	2	3	4	5	6	7
m	2,30	2,26	2,22	2,28	2,14	2,10	2,6
M	3,30	3,34	3,38	3,42	3,46	3,50	3,54
$M : m$	$\frac{7}{5}$	$\frac{107}{73}$	$\frac{109}{71}$	$\frac{37}{23}$	$\frac{113}{67}$	$\frac{23}{13}$	$\frac{13}{7}$

The analogy between Vettius Valens and Mich. Pap. 149 is striking, the only differences being in systems A and B on the one hand, and in the selection of fundamental latitude Alexandria and Babylon on the other. But also Vettius Valens calls the clima of Babylon the "second" [41] and both schemes define the climata by a constant difference of 4° as (13) p. 256 and (16) show, and as Vettius Valens explicitly states.[42]

System A for Babylon ($M : m = 3 : 2$) is once more followed by Manilius and by Bishop George,[43] for Alexandria ($M : m = 7 : 5$) in Hypsicles,[44] showing how familiar those "linear methods" were to Greek astronomers.

§ 4. Firmicus Maternus

11. In the second book of his "mathesis," Firmicus Maternus [45] gives the following list of rising times:

clima	Alexandria and Babylon	Rhodes	Hellespont	Athens	Ancona	urbs
$\alpha_1 = \alpha_{12}$	20	19	17	18	15	17
$\alpha_2 = \alpha_{11}$	24	23	22	23	21	22
$\alpha_3 = \alpha_{10}$	28	27	27	27	27	27
$\alpha_4 = \alpha_9$	32	32	32	32	32	32
$\alpha_5 = \alpha_8$	36	36	37	36	38	37
$\alpha_6 = \alpha_7$	40	40	42	41	44	42

The disorder is obvious, except in the first clima which is identical with the scheme A for Babylon. Honigmann recognized correctly [46] that the numbers can be explained by the assumption that the fractional parts are ignored, and he restored the following original list:

clima	[I]	II	III	IV	V	[VI]	VII
$\alpha_1 = \alpha_{12}$	21;40	20	19;¦26,40	18;¦53,20	17;¦30	16;40	15
$\alpha_2 = \alpha_{11}$	25	24	23;¦40	23;¦20	22;¦30	22	21
$\alpha_3 = \alpha_{10}$	28;20	28	27;¦53,20	27;¦46,40	27;¦30	27;20	27
$\alpha_4 = \alpha_9$	31;40	32	32;¦ 6,40	32;¦13,20	32;¦30	32;40	33(!)
$\alpha_5 = \alpha_8$	35	36	36;¦20	36;¦40	37;¦30	38	39(!)
$\alpha_6 = \alpha_7$	38;20	40	40;¦33,20	41;¦ 6,40	42;¦30	43;20	45(!)
M	3,30	3,36	3,38	3,40	3,45	3,48	3,54

[41] Vettius Valens I, 14 ed. Kroll p. 28, 24. [42] Vettius Valens I, 7 ed. Kroll p. 24, 17 f.

[43] Manilius Astron. III, 275 ff.; Ryssel [1] p. 47 f.

[44] Hypsicles, Anaphorikos, cf. note 40. Quoted also by Ptolemy, Tetrabiblos, I, 20 (ed. Robbins p. 94/95) = I, 21 (ed. Boll-Boer p. 46). [45] Firmicus Maternus II, 11 ed. Kroll-Skutsch p. 53 ff. [46] Honigmann [1] p. 45.

He had to assume the omission of [I] and [VI], the duplication of V and three errors of one unit each in VII. Moreover, this scheme has not the fundamental property of linear increase of maximal lengths of the days from clima to clima as found in Vettius Valens and Pap. Mich. III, 149.

There exists however, still another possibility, which, omitting the fractional parts, gives the numbers of Firmicus exactly without any correction and explains the duplication of the column beginning with 17. We saw that Vettius Valens started his list from Alexandria, increasing the value of M by 4 from clima to clima; on the other hand, Pap. Mich. III, 149 began from Babylon, but with the same difference in M (and using system B instead of A). The sets of M-values subdivide each other symmetrically, as the following comparison shows

(*a*) Vettius Valens:	3,30		3,34		3,38		3,42		3,46		3,50		3,54	
(*b*) Mich. Pap. III, 149:		3,32		3,36		3,40		3,44		3,48		3,52		3,56

If we combine these two schemes (of course replacing system B in the second line by A) we get a scheme which contains the following tables:

climata	IIb	IIIa	IIIb	[IVa]	IVb	Va	[Vb]	[VIa]	VIb
$\alpha_1 = \alpha_{12}$	20	19;26,40	18;53,20	18;20	17;46,40	17;13,20	16;40	16; 6,40	15;33,20
$\alpha_2 = \alpha_{11}$	24	23;40	23;20	23	22;40	22;20	22	21;40	21;20
$\alpha_3 = \alpha_{10}$	28	27;53,20	27;46,40	27;40	27;33,20	27;26,40	27;20	27;13,20	27; 6,40
$\alpha_4 = \alpha_9$	32	32; 6,40	32;13,20	32;20	32;26,40	32;33,20	32;40	32;46,40	32;53,20
$\alpha_5 = \alpha_8$	36	36;20	36;40	37	37;20	37;40	38	38;20	38;40
$\alpha_6 = \alpha_7$	40	40;33,20	41; 6,40	41;40	42;13,20	42;46,40	43;20	43;53,20	44;26,40
M	3,36	3,38	3,40	3,42	3,44	3,46	3,48	3,50	3,52

Except for the omission of three climata (and the restriction to integers) we have here exactly the numbers given by Firmicus Maternus. We do not even need to speak about "omissions." What Firmicus (or, better, his source) did was simply to take a list of rising times, progressing by 2° in maximal length of the days, and to select those groups which corresponded to seven famous cities.

12. Having achieved this insight into the simple structure of lists of rising times, i.e. for the purpose of arranging geographical zones according to linear increasing values of M, we now have to discuss the names associated with these zones. Obviously, much disorder prevails here, but it seems to me still possible to reach some understanding at least of the lists found in our sources. We saw that the leading principle in the order of all the tables of rising times is their arrangement according to increasing maximum length of the daytime. Taking this as granted, Firmicus' series of place-names becomes correct as far as the increase of latitude is concerned: Alexandria → Babylon → Rhodes → Athens → Hellespont → Rome → Ancona. The same is true in the case of Mich. Pap. III, 149 with the only exception of the totally misplaced "Argos":[47] Ethiopia → Syria → Rhodes → Asia and Ionia → Argos(!) → Rome, Italy, Gallia maritima → As[ia], Germany, Britain. If we remove Argos and replace it by Rome, accepting Italy and Gallia as equivalents of Ancona, then agreement between Firmicus and the Mich. Pap. is reached from Babylon to Ancona.[48] The papyrus adds as southernmost clima Ethiopia, as northernmost Germany and Britain, doubtless incorrect as far as the latitudes are concerned, but obviously taken from some scheme considering these zones as boundaries of the oikumene.[49]

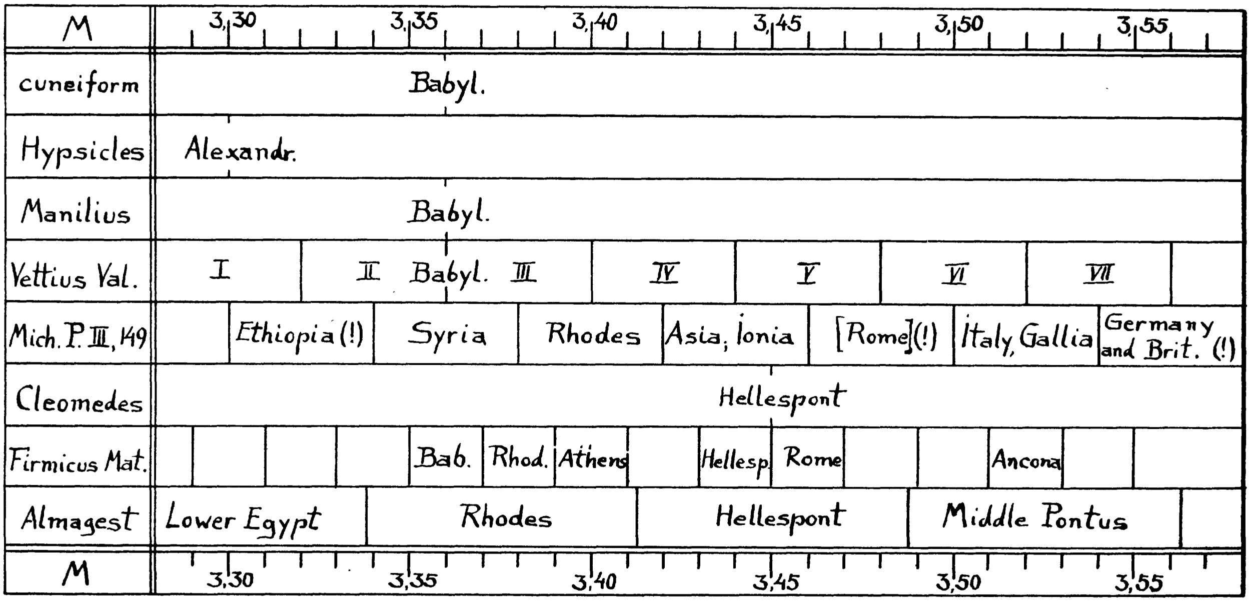
Serious difficulties occur if we consider the numbers in each table as exact representatives of the latitude of the locality heading the column. However, there exists no proof of the correctness of this assumption. We may just as well assume that the values given refer to the central line in a certain zone to which the place in question belongs, without lying exactly in the middle of the strip, the extent of each strip being determined simply by the regular increase of M from clima to clima.[50] The resulting arrangement is repre-

[47] Honigmann (M. P. III p. 304 note 10) assumes tentatively Argos Amphilochensis which in itself seems to be very unlikely and makes the arrangement no better than if we assume the Peloponnesian Argos.

[48] We therefore do not need to accept Honigmann's "überraschendes Ergebnis, dass die Benennungen der sieben Klimata in dem Papyros völlig aus der Luft gegriffen sind, mit ihnen nichts zu tun haben und von uns unbeachtet gelassen werden dürfen!" (M. P. III, p. 304).

[49] Ptolemy opera II p. 160, 2 ff. (προχ. καν.) speaks about τούτων μεταξύ πως τῆς οἰκησίμου παραλλήλων ἑπτὰ παρακειμένων. For Ethiopia on the one hand and the Ister on the other as boundaries of the oikumene see e.g. Heidel [1] p. 26 ff. and p. 31 ff.

[50] Diller [1] p. 264 came to the same result from absolutely different considerations.



sented by the figure above. Except for the three above-mentioned errors in Mich. Pap. III, 149, no further correction of the tradition is necessary. The zones overlap each other in such a way that the same place names have always at least parts of the zones in common.

13. Honigmann drew attention to another source of information about the anaphorai,[51] namely the astrological calculation of the maximal possible length of the human life. This is the doctrine that the number of years can never exceed the maximal possible number κ of degrees which is necessary for one quarter of the ecliptic to rise. This number is obviously given by

$$\kappa = \alpha_5 + \alpha_6 + \alpha_7 = \alpha_6 + \alpha_7 + \alpha_8$$

and, according to Epigenes, has the value 112 = 1,52; according to Berossos, 116 = 1,56; and "in Italiae tractu," 124 = 2,4.[52] Honigmann saw that the difference between these values must correspond to the difference in the respective clima and that Epigenes speaks about Alexandria, Berossos about Babylon. We can now formulate these results a little more precisely by calculating κ according to the two systems the existence of which we recognized in simultaneous use. We obtain for κ:

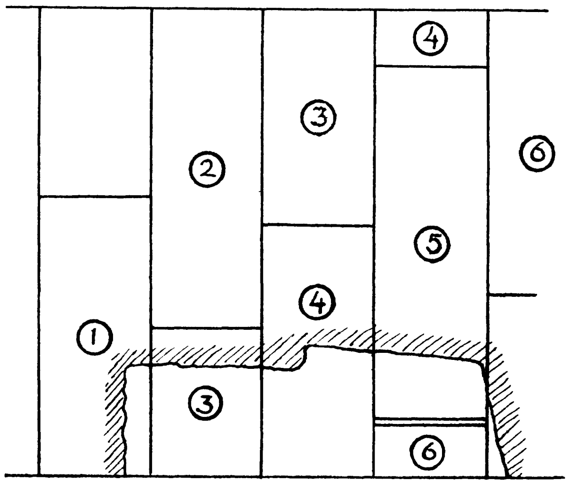
κ	A	B
Alexandria	1,51;40	1,50
Babylon	1,56	1,54

[51] M. P. III p. 307 ff. The necessary references are given there.
[52] Pliny N.H. VII, 160 (ed. Mayhoff II p. 56, 2).

This shows that the values given by Epigenes and Berossos are both based on system A. As to the value κ = 2,4 "in Italiae tractu," we find from Mich. Pap. III, 149 column 6 in (12) p. 256 the value 2,4;40 and this is just the column headed as "Rome, Italy, Gallia maritima." This shows again the correctness of the place names in the text.

§ 5. Pap. Mich. III, 151

14. A more elaborate type of obliquascensional table is preserved in Pap. Mich. III, 151, although in such a fragmentary condition that I did not succeed in giving a complete restoration. The essential point is that this text gives the rising times not only from sign to sign, but from degree to degree, and furthermore, the first degree is subdivided into $\frac{1}{3}$ (= 0;20) and $\frac{2}{3}$ (= 0;40). The fragment contains parts of tables for five different zodiacal signs. The arrangement of these tables, however, is not such that each table



occupies one column, but table follows table as the height of the text permits. The previous figure explains this situation. A simple calculation shows that each column contained 46 lines from which only the lower quarter has been preserved.

In order to discuss the structure of these tables in detail, it is preferable to start with section 6, where the title *ταυρου διδυμ[ου]* is preserved. That two signs are mentioned may be explained by assuming the vernal point somewhere inside the sign Aries (e.g. 8° or 10°) such that α_2 belongs to both ♉ and ♊. The numbers given in the text are: [53]

section 6:	0,29,56	0,20
	0,59,52	0,40
	0,22,48	1
	2,59,36	2
	4,29	3
	5,59	4

If we take 0;29,56 as $\frac{1}{3}$ of some unit, as indicated by the numbers in the second column, we obtain

0;29,56	0;20
0;59,52	0;40
1;29,48	1
2;59,36	2
4;29,24	3
5;59,12	4

This indicates that in line 3 the papyrus has the erroneous figure 0,22 instead of 1,29 and that the last place in the last two lines is omitted. That this omission was intentional is shown by section 3: [54]

section 3:			calculation:	
1.	[0,27 ,10]	[0;20]	0;27,10	0;20
	[0,54 ,20]	[0;40]	0;54,10	0;40
	1,21[,30]	1	1;21,30	1
	2,43	2	2;43	2
5.	4,30	3	4; 4,30	3
	9,26	4	5;26	4
	6,47	5	6;47,30	5
	8, 9	6	8; 9	6
	10,52	8	9;30,30	7
10.	12, 3	9	10;52	8
	13,35	10	12;13,30	9
	14,56	11	13;35	10
			14;56,30	11

Here, the text omits all 30's except in line 5, where 4,30 is an error for 4,4,30.[55]

We are now able to restore section 4. Corresponding to 20 the text gives the number 30,36 and therefore the number corresponding to 1 must be 30,36 : 20 = 1,31,48. We therefore should have

19	29, 4,12
20	30,36
21	32, 8,48

but the text gives 29,13 corresponding to 19 and 32,18 corresponding to 21. However, if we correct 30,36 to 30,46, we obtain full agreement, except, of course, for the omission of the third sexagesimal place: [56]

section 4:			calculation:	
1.	24,36	16	24,36,48	16
	26, 9	17	26, 9, 6	17
	27,41	18	27,41,24	18
	29,13	19	29,13,42	19
5.	30,36	20	30,46	20
	32,18	21	32,18,18	21
	33,50	22	33,50,36	22
	35,22	23	35,22,54	23
	35,55	24	36,55,12	24

15. Hence, we have reached a complete understanding of the arithmetical structure of sections 3, 4 and 6. Each section is an arithmetical progression δ, 2δ, 3δ, $\cdots$, 30δ, adding $\frac{1}{3}\delta$ and $\frac{2}{3}\delta$ at the beginning, and in general omitting the third place of all numbers. Each section is therefore completely characterized by one number, say 30δ, as follows:

section:	3	4	6
30δ:	40;45	46;9	44;54.

We know, furthermore, that section 6 refers to the signs ♉ and ♊, and it is obvious that the numbers 1, 2, $\cdots$, 30 refer to degrees. Such lists would be the analogue of the *κανόνιον τῶν κατὰ δεκαμοιρίαν ἐπισυναγόμενοι* in Ptolemy Almagest II, 8 giving the "*χρόνοι ἐπισυναγόμενοι*" but from degree to degree. The only objection to this explanation of our tables seems to be the magnitude of the numbers involved: rising-times of more than 40 degrees are possible only in the northernmost climata and then never for signs around ♉. This can however easily be explained by the choice of units. As mentioned above p. 254 Manilius expresses the obliquascensional times in "stadia" and one stadium equals two degrees. If we assume the same units to be applied here, then the right order of magnitude is restored. 44;54 stadia = 22;27° is a perfectly possible value for α_2 (e.g. in the fifth clima $\alpha_2 = 22°$ in Mich. Pap. III, 149 and $\alpha_2 = 22;20$ in Vettius Valens and $\alpha_2 = 22;30°$ on the Hellespont according to Cleomedes).[57]

We must still consider section 5. Two lines only are preserved: [58]

52,45	29
54,13	30

[53] Line 4: *β νθ λς* instead of *β νδ λς* as given in the edition according to friendly investigation of the original by Dr. F. E. Robbins.

[54] Line 3: *α̣ κ̣α̣* Robbins.

[55] Real errors are: line 6: *θ κγ* instead of *ε κγ*. Omission of the line *θ λ λ|ξ*. Line 10: *ιβ γ* instead of *ιβ ιγ*.

[56] One more error in the last line: *λε νε* instead of *λς νε*.

[57] Cf. p. 257.

[58] Remainders from the preceding line: *ν̣* at the first place (Robbins).

and in the second line 54,13 could be replaced by 54,16 (i.e. γ by ς). However, at least one of these two lines must contain an error, because from $30\delta = 54$ follows $\delta = 1,48$, but the difference between 52,45 and 54,13 or 54,16 is only 1,28 or 1,31. A simple consideration of all possibilities of correcting this error shows that by far the most plausible assumption is the replacement of 40 by 50, leading to the following values in section 5:

$$1\delta = 1,28,27$$
$$29\delta = 42,45,\ 3$$
$$30\delta = 44,13,30$$

from which the text omits, as usual, the third sexagesimal place.

This finally would give us the following set of α's, contained in Mich. Pap. 151:

section	3	4	5	6
α	20;22,30	23;4,30	22;6,45	22;27

All these values are possible for α_1 and α_2 in the usually considered climata but I cannot find any relation which could define them exactly. The differences between these α's do not agree with the assumption of one geographical latitude. The only conclusion left, as far as I can see, is the following: sections 3 and 4 give the values for α_1 and α_2 for a clima about Rhodes. Sections 5 and 6 however both give α_2 but for two special places, about $1\frac{1}{2}$ or 2 degrees different in latitude, somewhere between Rhodes and the Hellespont.

§ 6. Conclusions

16. The representation of the anaphorai of the zodiacal signs by means of simple arithmetical series is an interesting example of how problems can be solved which seem to require trigonometrical methods. The anaphorai, however, are only one example of the treatment of periodic phenomena by linear approximations (or iterated linear approximations) which was developed to a surprising degree by Babylonian astronomy of Seleucid times and has gained the highest admiration of every scholar who studied this subject.

Having adopted the type of approximation, either A or B as previously designated, the anaphorai for all zodiacal signs are determined by one single constant M, the length of the longest day, or by the equivalent ratio $M : m$ assumed to be 3 : 2 in Babylon. It is interesting to realize that this method was not merely adopted by the Greeks for special latitudes like Babylon or Alexandria, but consistently expanded as soon as the discovery of the sphericity of the earth required the consideration of changing horizons.[59] The Greek definition of the climata by a linear sequence of values of M instead of by the geographical latitude [60] shows exactly the same methodological idea as the Babylonian astronomy. These methods furnished the empirical material for the numerical determination of astronomically important magnitudes, and prepared the ground on which trigonometrical methods could be developed step by step. It is not surprising that these methods were even used long after the definitive creation of spherical trigonometry, because of their simplicity, not requiring elaborate tables.

It seems unnecessary to me, however, to assume the existence of any uniquely determined Greek doctrine of the climata, or a clear distinction between "astrological" and "astronomical" climata as Honigmann does. The preserved texts show the existence of tables of rising times (or equivalent tables of length of the days) for special places, like Alexandria, Babylon or the Hellespont, having in common only the choice of a convenient simple value for $M : m$. On the other hand, the tables for groups of climata (as Vettius Valens or Mich. Pap. III, 149) have no more in common than the general arrangement of the climata according to linear increasing values of M, but differ in the choice of the starting point (Babylon or Alexandria) and in the system adopted (A or B).

17. As stated previously, the Babylonian origin of the "linear" theory of the anaphorai is obvious. More difficult is the determination of the time of adoption of the basic ideas. The Babylonian methods of calculating the movement of the sun and the moon and the planets from which we have our information about the two systems A and B cannot be traced much further back than 200 B.C. The oldest dated tablet, although belonging to the later system (B), refers to the year 208 B.C.[61] Our present source material, very complete for the period from 208 B.C. to 46 B.C., does not require the assumption of a date for the highest level of Babylonian theoretical astronomy earlier than, say, 250 B.C. We know, on the other hand, that the contact of Greek and Babylonian astronomy had its climax about 200 to 150 B.C.[62] This would fit in very well with the fact, pointed out especially by Honigmann, that the theory of the seven climata belongs to Eratosthenes.[63]

The restriction of the number of "climata" to seven marks neither the beginning nor the end of the

[59] That this happened later than in the fifth century, seems to me now definitely established by the researches of Frank ([1] p. 184 ff.) and Heidel ([1] p. 63 ff. esp. p. 79 f.).

[60] Cf. p. 255.

[61] The oldest text preserved is Chicago A 3430 + A 3431 (to be joined with Warka X 63, unpublished, No. 125 of my edition of cuneiform texts under preparation). Schnabel's attempts to date both systems ([1] p. 7 ff. and [2] p. 218 f. and p. 223 ff.) are based on wrong assumptions. Also Olmstead's attempt ([1] p. 122 f.; the quotation in note 30 is wrong; replace O. Schroeder by A. Ungnad) is inconclusive, because there is no proof that the Nabu-rimanni, who witnessed documents in 491/0, is the astronomer of system B whose name appears in a tablet of 46 B.C. (VAT 209 = No. 16 of my edition; published with many errors by Schnabel [2] p. 244 f.).

[62] Cumont [1].

[63] Honigmann [1] p. 10 ff. Cf. furthermore Diller [1] p. 263.

Greek concept of geographical zones. Ptolemy ignores the restriction to seven both in the φάσεις and in the Geography,[64] and the relationship of Epigenes' and Berossos' numbers of the longest possible lifetime to the anaphorai [65] supposes the existence of the Babylonian scheme "A" at least around 300 B.C.[66] This does not contradict our statement that Babylonian theoretical astronomy scarcely originated before 250 B.C. From Vettius Valens we know of attempts of undoubtedly Babylonian origin to determine the relation between the visibility of the new crescent and the anaphorai of the zodiacal signs.[67] This theory is much more primitive than the elaborate method known from moon-theory A. On the other hand, there exist much older cuneiform tablets which attempt to describe the relation of the moon's invisibility to the seasons by still rougher linear approximations.[68] This shows that the problem of the determination of the length of the days belongs to the oldest part of Babylonian astronomy, preceding the theory of the planetary movement and the theory of the moon of the Seleucid and Arsacid period in the same sense as these linear methods discussed here preceded the later Greek sphaeric.

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[64] Cf. note 49 p. 259 and note 26 p. 255.

[65] Cf. p. 260.

[66] Honigmann M. P. III p. 310 f.

[67] Vettius Valens I, 14; ed. Kroll p. 28.

[68] I intend to discuss this material from cuneiform sources separately in a forthcoming paper.

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